Co-funded by the European Union

MODULE 06

TRAINING PROGRAMME

GEOLOCATION

Table of contents

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1. Objectives of the module

This module describes coordinate systems used to locate points on earth. It explains how a map is made in a projection system. It also helps to understand how to geolocate terrestrial observations.

In order to locate information on the Earth's surface, it is necessary to use a positioning and mapping system. For this, geodesy notions are necessary, such as:

- the definition of a geodetic reference frame (ellipsoid, meridian of origin)
- the choice of a system of projections and coordinates (geographic or planar)
- the choice of a height reference system (geoid).

The module explains also how a Global Navigation Satellite System (“GPS”) works and how to get a centimetric accuracy.

At the end you'll see how to position targets on the field to geolocate drone observations and as a result a point cloud.

2. Modelling the terrestrial globe

2.1 The ELLIPSOID: modelling the earth by a perfect geometric surface

As seen from space, the Earth has the shape of a sphere, but in fact, it is slightly deformed by the centrifugal force induced by its rotation around the pole axis. The topographic model of the Earth is close (to a few meters outside of the relief) to a known mathematical volume: the ellipsoid of revolution (rotation of an ellipse around the axis of the poles (the minor axis)).

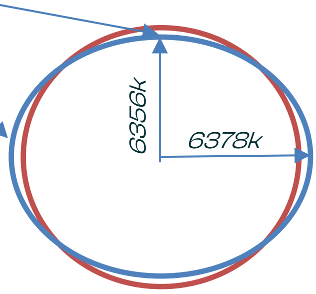
Average radius of: 6 367 km

Settling at the poles: -11 km compared to the sphere

Bulge at the equator: +11 km compared to the sphere

Red : a circle (2D), a sphere (3D)

Blue : an ellipse, an ellipsoid (3D)
= the general shape of the earth



Order of size for an ellipsoid of revolution of the Earth :

- a = 6 378 137m = equatorial radius of the earth

Figure 2-1 WGS84 ellipsoid

b = 6 356 752 m = polar radius of the Earth

Equatorial circumference = 40 074 km

Length of a meridian ellipse = 40 007 km (integral calculation)

To make topography we project on the ellipsoid the contours of the elements to map.

2.2 International reference Ellipsoid: WGS84

WGS 84 (World Geodetic System 1984); the International Reference system, is the geodetic system defined by scientists as the ellipsoid IAG GRS 80 (with 0.1mm modification) and the geoid (see below) EGM96. It became inescapable now because it is the reference used by the GPS satellite positioning system and all other systems (European Galileo, Russian Glonass, Chinese Beidou, Japanese KZSS and Indian IRNSS). It has quickly become the universal reference for cartography (marine and land).

The local ellipsoids defined in general for each country, tend to disappear in favour of WGS84. They date from the time when scientists were not all in agreement worldwide, some countries have kept and may differ from WGS84 to the point of distorting the coordinates. For example, the old French maps from before 1999 which were in Clarke 1880 ellipsoid are not "GPS compatible".

➔ It is therefore always necessary to check whether the projection ellipsoid is WGS84 (or IAG GRS80). Any other reference ellipsoid would cause a positioning error.

2.3 Geographic North and magnetic North

Geographic North = axis of rotation of the earth = attachment of meridians = TRUE North = North of the maps

THIS IS THE ONLY NORTH WE USE IN TOPOGRAPHY

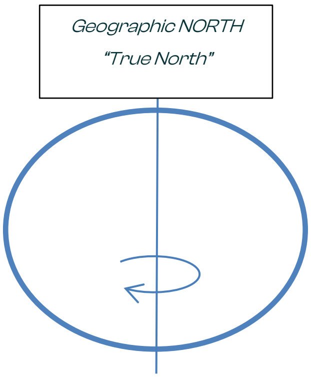


Figure 2-2 rotation axis = geographic North (right)

For information:

Magnetic North = point where the earth's magnetic field "enters" the earth: the field lines meet. It is close to the geographic North, but not coincident and variable, it cannot be used as a reference.

The compass indicates the magnetic North, this direction is different from the geographic North by the declination: angle between the North direction indicated on a compass and the "true" North direction, geographic.

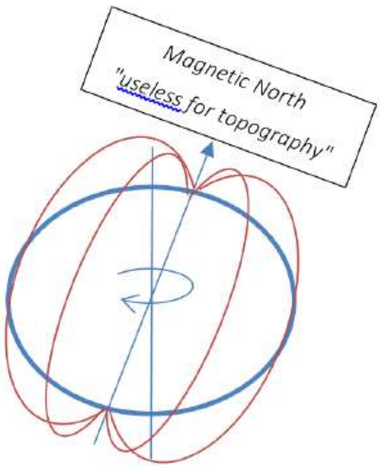


Figure 2-3 magnetic field declination in 2015 (ncei.noaa.gov)

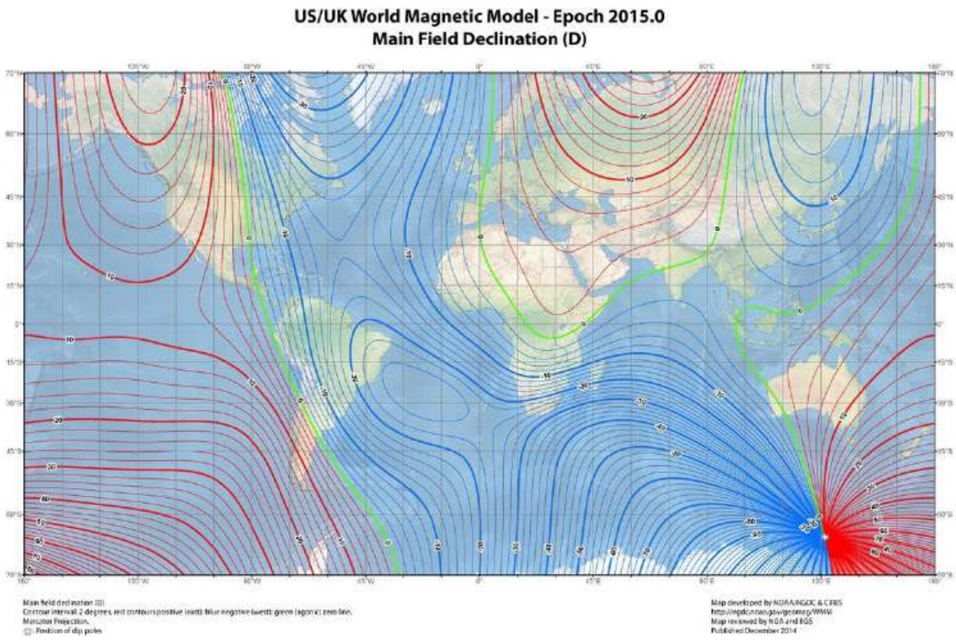


Figure 2-4 declination of magnetic field lines of the earth

In France the declination is low (1 to 3°), but in Poland it reaches 6°!!

A compass is not a positioning instrument giving the North of the maps, but the local declination is indicated on the maps to allow the use of a compass for hiking or approximate navigation.

3. Positionning on the ellipsoid

3.1 Geographic coordinates: longitude, latitude, ellipsoidal height, international

The three-dimensional geographic coordinates (λ,ϕ,h) of any point, allow to position this point on the ellipsoid and to give its elevation from this ellipsoid.

They are based on two references:

The **longitude** l (lambda) which is the angle formed by the **meridian** passing through the point considered with the prime meridian.

The international prime meridian is that of Greenwich (observatory in London).

The international reference meridian passes through the Greenwich Observatory in London, and in France around Tarbes, 60km west of Toulouse, around Angoulême and around Caen.

The **longitude** is between 0° and 180° towards the East, we say "East longitude" or between 0° and 180° towards the West, we say "West longitude".

Figure 3-1 meridians (brainly.in)

The **latitude** f (phi) which is the angle that the normal to the ellipsoid makes with the plane of the equator (= equatorial plane) determining the **parallels**.

When the **latitude** is between 0° and 90° towards the North, it is a "North latitude" or between 0° and 90° towards the South, it is called a "South latitude".

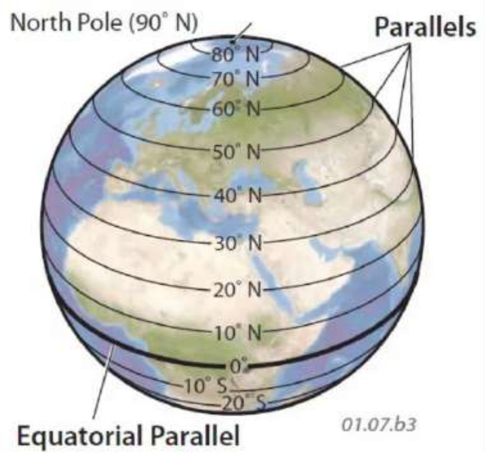


Figure 3-2 parallels (brainly.in)

(https://geography.name)

Examples of European partners coordinates (from W to E)

Cork, Munster Technological University: 51.885808N ; 8.533495W

Madrid, CRN Paracuellos: 40.494217N ; 3.535192W

Nîmes, Campus Herec: 43.827286N ; 4.356369E

FH Kufstein Tirol: 47.583670N ; 12.173542E

The **ellipsoidal height h** is the distance between the point considered and the projection of this point on the ellipsoid.

It is this ellipsoidal height that satellite systems determine, and not the usual elevation (above sea level) which they do not know...

NB: the measure of “h” is orthogonal to the ellipsoid: so it does not reach the centre of mass of the earth see the gap here

So as latitude angle is not reaching the centre of mass...

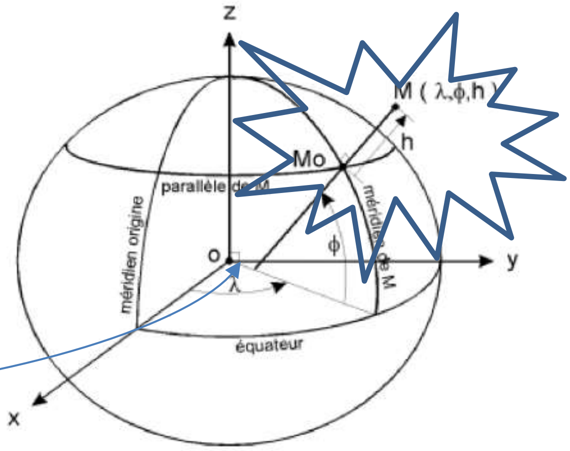


Figure 3-3 ellipsoidal height orthogonal to the ellipsoid

In the illustration, the point Mo is the projection of M perpendicular to the ellipsoid.This allows it to be located with latitude and longitude.

All countries in the world use the same geographic coordinate system (even North Korea!).

3.2 Local sphere approximation

For some calculations, it is useful to replace the ellipsoid by a sphere with a radius locally adapted to the ellipsoid.

In France the sphere has a 6380km radius

3.3 Geodetic Datum

The characteristics of the ellipsoid and the positioning parameters constitute a geodetic datum.

A geodetic datum is therefore defined by :

- An ellipsoid → WGS84 (or call it IAG GRS 80)
- The position of the centre of the ellipsoid according to the terrestrial equator position known at a few meters.
- The orientation of the axes of the ellipsoid → major axis along the equator, minor axis towards the axis of rotation of the earth
- A meridian origin of longitudes → the meridian passing through the observatory of Greenwich in London
- A plane of origin of latitudes → the plane of the equator

3.4 Cartesian coordinates: ITRS89 (world) and ETRS89 (Europe)

Instead of geographic coordinates, a fixed Cartesian reference frame centred on the centre of mass of the earth is defined worldwide: the International Terrestrial Reference System (ITRS) since 1989.

These are three coordinates: X, Y, Z where:

- The X,Y plane is the equator
- The Z axis is the axis of rotation of the earth
- The point 0,0,0 is the centre of mass of the earth
- The X axis passes through the meridian of Greenwich

Each point on the earth can have coordinates (X,Y,Z)

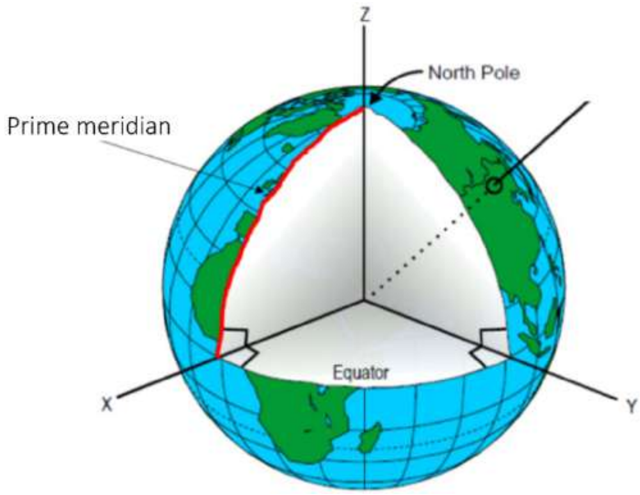


Figure 3-4 axis of Cartesian coordinates (researchgate.com)

Problem: continental drift…

Space geodetic systems (i.e. modern ones) are very accurate, and allow to express in the same system the coordinates of points located on different tectonic plates: the relative movements of these plates (up to several cm/year) cannot be neglected anymore… The ITRS (International Terrestrial Reference System), which is the most accurate geodetic system in the world (centimetric accuracy), is constantly evolving; each of its realizations (ITRF, for International Terrestrial Reference Frame), consisting of a network of ground stations whose coordinates and displacement speeds are fixed, is dated: the ITRF90 corresponds to the value of these elements for the year 1990. Actually, the last computation is ITRS2020 (https://itrf.ign.fr/en/solutions/itrf2020)

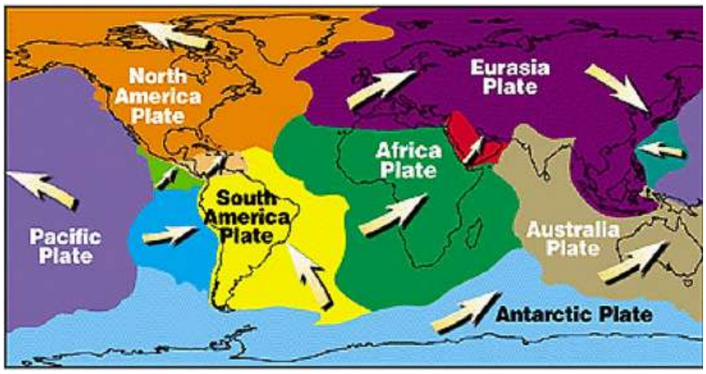


Figure 3-5 continental drift (importantinnovations.com)

This international system is declined in Europe under the name of ETRS 89, with a network whose latest version is ETRF2020. In Europe, it was decided to attach our reference system to the Eurasian plate in 1989: from that year on, the ETRS (European terrestrial reference system) differs from the ITRS because of the shift of the continental plate to which it is attached. The transformation parameters must be updated all the time ETRS→ITRS

https://epncb.oma.be/_productsservices/coord_trans/

The ETRS is used as a basis for all plane projection systems of European countries, its ellipsoid is GRS80 (=WGS84 at 0.1mm) and there are more than 300 measuring stations in Europe (EUREF).

We do not use these coordinates for the moment with drones, but it would be possible.

4. Position points on a map, a plan

4.1 Double projection for a map

A plan or a map is the representation of a part of the ellipsoid on a developable surface, on a surface that can be spread out flat: essentially the cylinder, the cone and the plane.

To make a map or any terrain plan, it is necessary to make two projections:

- The real **terrain**, the measures, are projected **on the ellipsoid**
- The shape on the **ellipsoid** is then projected on a **developable surface** to obtain a planar representation

Thus, angles, distances, areas measured on the maps may differ a lot from the one measured...

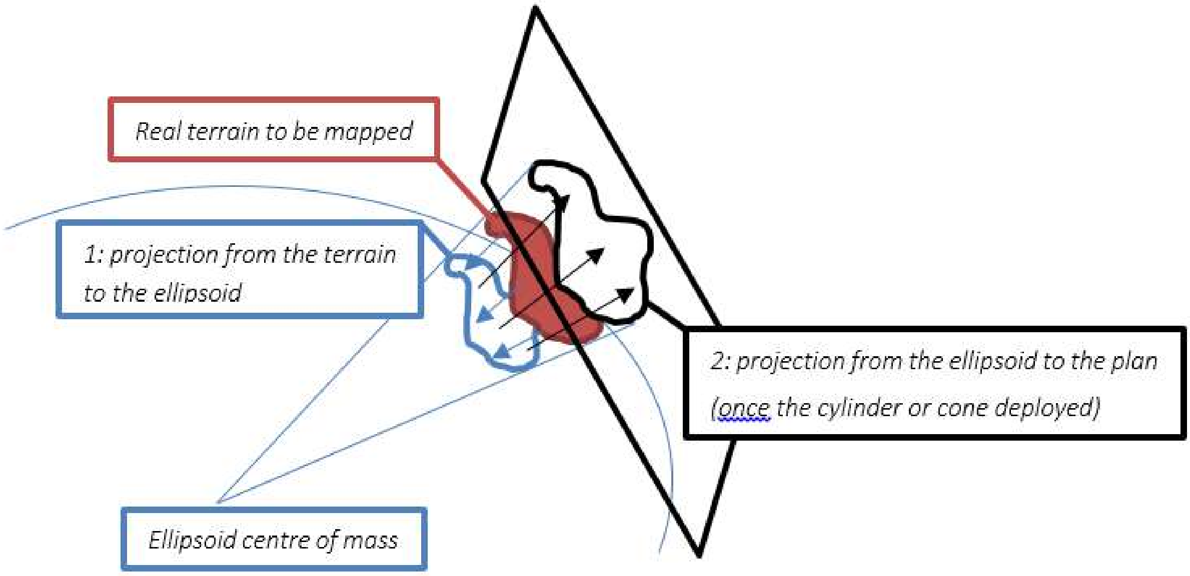


Figure 4-1The two successive projections to obtain a map

- The projections used for topography are **conformal**: they locally preserve the angles. An angle measured on the field is the same as on the plane.
- The projections **alter the distances** (modify the distances). A distance measured on the ground is not the same as the one measured on the map, even if the map scale is applied.
- In these projections we define an **orthonormal frame of reference**, oriented positively towards the East and the North, specific to each projection system, which replaces the Longitude, Latitude; coordinates by X, Y or E, N .

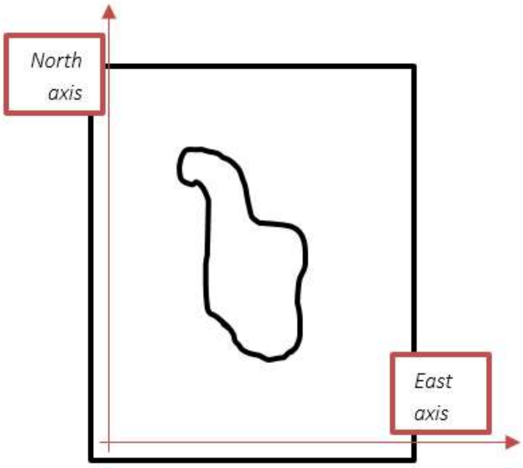
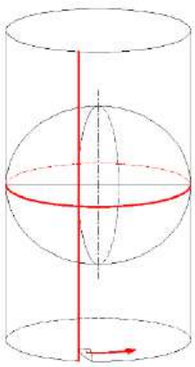
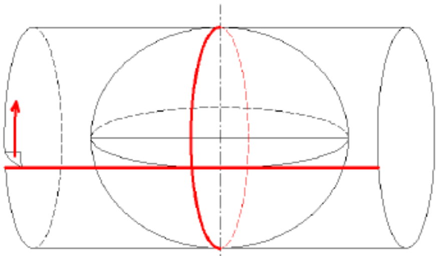


Figure 4-2 orthogonal frame of reference on a planar map

The cylindrical projections

The projection surface is a cylinder circumscribed along the equator ("direct") or a meridian ("transverse") of the ellipsoid (example: UTM, Gauss, ...).

 *Figure 4-3 direct cylindrical projection*	Direct cylindrical representation: used for large maps showing several countries
 *Figure 4-4 transverse cylindrical projection*	Transverse cylindrical representation: used for maps and plans in many countries

The conical projections

Introduced by the mathematician Johann Heinrich Lambert (France) in 1772.

The projection surface is a cone tangent to a circle or secant to two circles (example: Lambert 93 in France, ...)

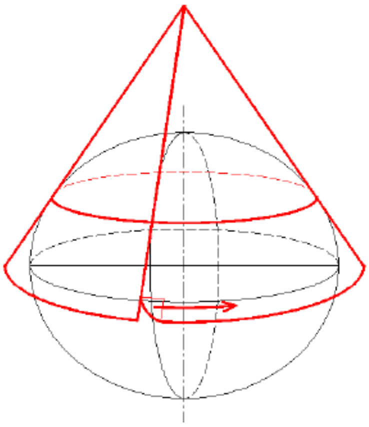


Figure 4-5 tangent conical direct representation representation

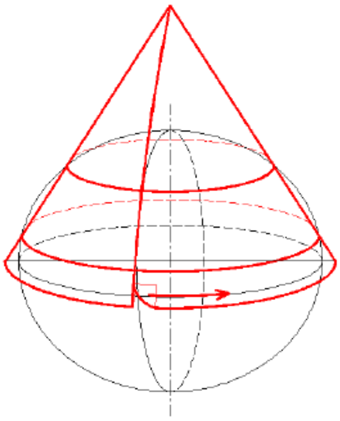


Figure 4-6 Direct secant conical

The azimuthal projections

The projection surface is a plane tangent to a point or secant to a circle

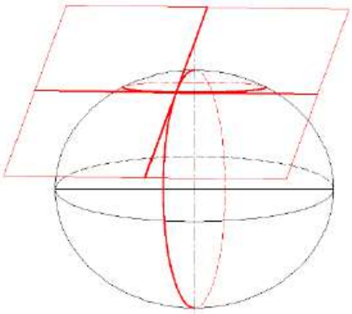


Figure 4-7 Azimuthal tangent representation

A projection that cannot be classified into one of these types is called individual or unique.

4.2 Projection and geodetic system

Do not confuse a projection with a geodetic system (allowing to locate a point on the surface of the earth).

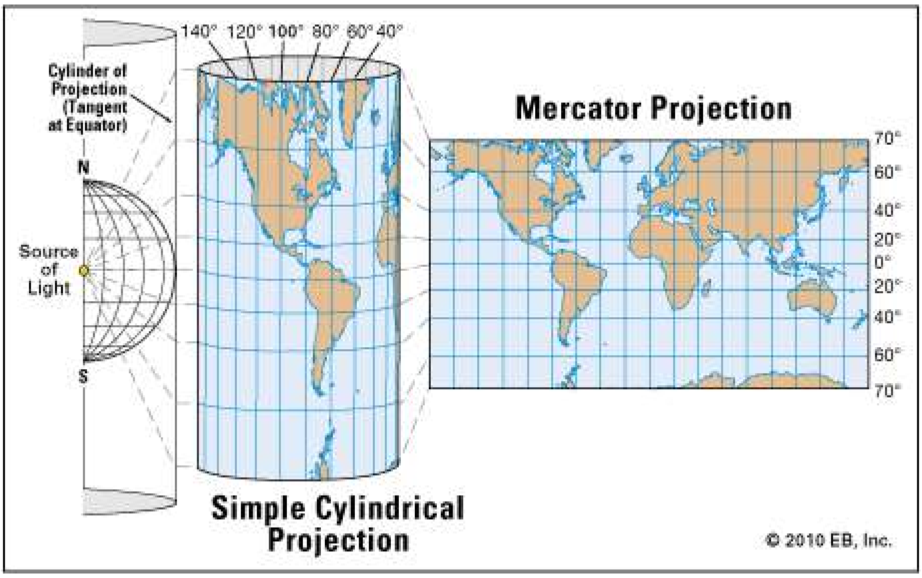
- ➔ The projection is the shape and position of the developable surface: cylinder or cone, contact with the ellipsoid.
- ➔ The geodetic system is the way of expressing the Cartesian coordinates on the projection: origin and orientation of the X and Y axes (or E and N)

Theoretically, any projection could be associated with any geodetic system, but to avoid ambiguities, we generally associate a geodetic system with a given projection (for example: ED 50 or WGS 84 with UTM projection, RGF 93 with Lambert 93 projection, ...).

Also, when giving the plane coordinates of a point (as the same projection can be used by several geodetic systems), it is necessary (except in obvious cases) to indicate both the projection and the geodetic system used (for example: E and N in UTM ED 50).

4.3 The Mercator projection

The ellipsoid is projected on a vertical cylinder



Equator = central line without distortion

Figure 4-8 direct Mercator projection (Britanica.com)

This results in a strong deformation when moving away from the equator. The equator is the central parallels or grid lines: only along the equator do the distances measured on the map correspond to those on the ground.

This is the distortion of distances: the distances measured on the map, even multiplied by the scale, are not equal to those measured on the ground… except along the equator.

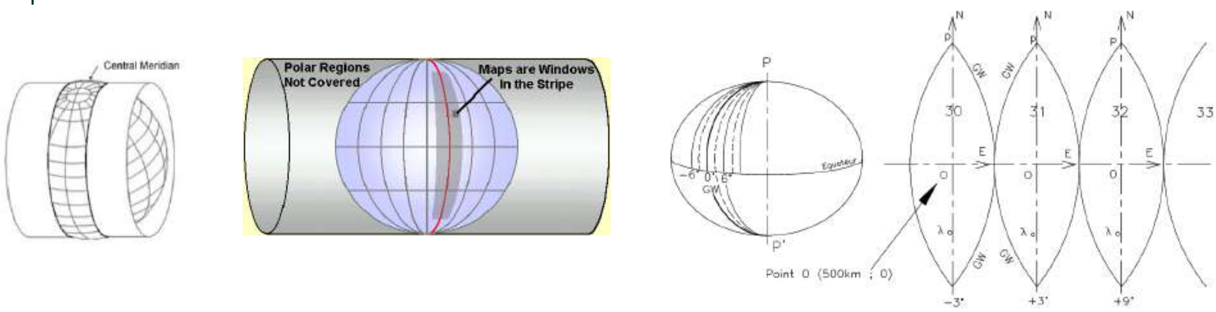
Widely used to represent the earth as a whole, or an entire country, in atlases.

Never used for civil engineering applications.

4.4 The projection and the system UTM: UNIVERSAL TRANSVERSE MERCATOR

UTM projection: transverse Mercator

The UTM projection (Universal Transverse Mercator defined around 1950 by the U.S. Army to represent the entire Earth) is a cylindrical transverse secant conformal projection covering the entire world in 60 zones of 6 degrees of amplitude in longitude (to limit the distance distortion at the edge of the spindles) now based on the WGS84 ellipsoid.



e-education.psu.edu oc.nps.edu unstats.un.org

Figure 4-9 Universal Transverse Mercator projection UTM

The meridians have as image on the plane curves depending on their distance from the central meridian. The meridian of Greenwich (= 0°) separates the 30 and 31 zones. The 2 secant meridians are called “standard meridians”: along them, no distance distortion.

The parallels have as image on the plane parallel curves equidistant to each other.

Conformal projection: same angles on ground and on map, but distance distortion.

The zones are numbered to identify them: the numbering of the zones starts at the 180° meridian and increases from West to East.

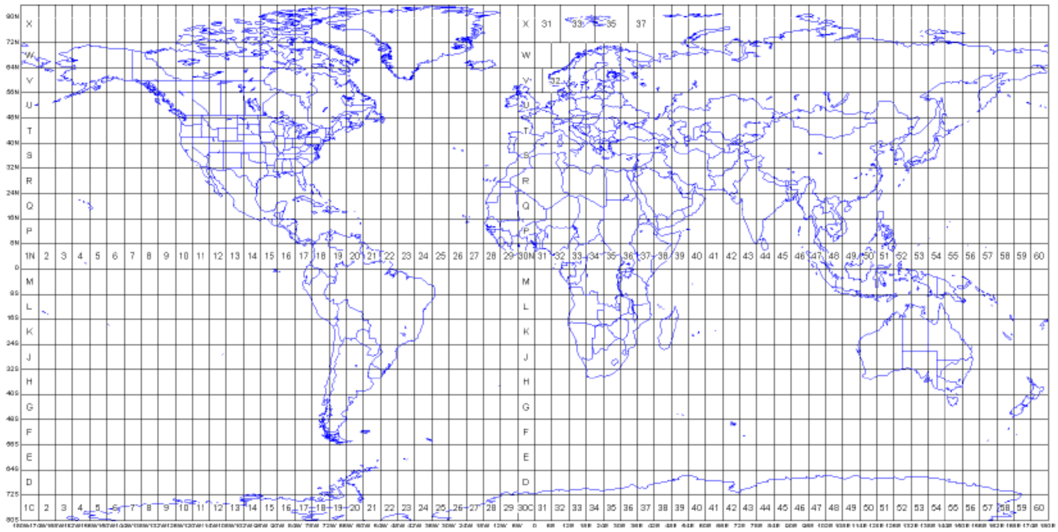


Figure 4-10 the 60 zones of the UTM (researchgate)

UTM coordinates system

On the map, the central meridian is represented by a straight line on the North axis.

All zones are identical.

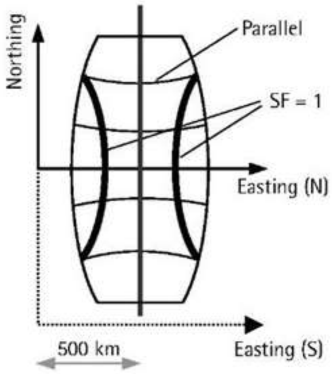


Figure 4-11 axis in a UTM strip

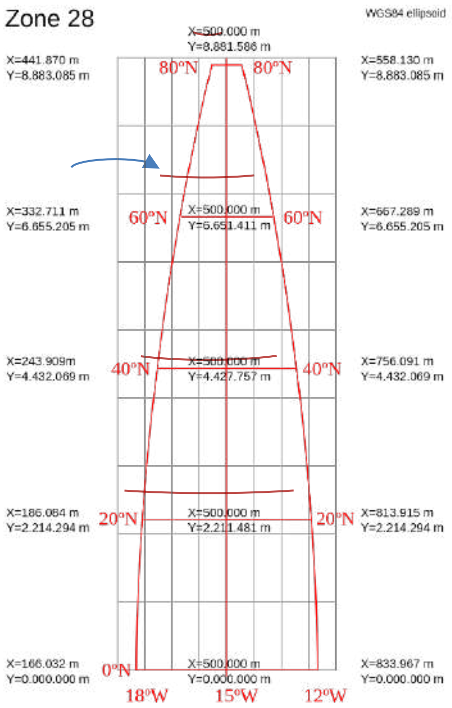


Figure 4-12 coordinates in a UTM strip

what-when-how.com

To avoid negative abscissas, the 0 point of each zone has the coordinate E0 = 500 km

The origin of the E and N axes has the coordinates :

- N_0 = 0 m in the Northern hemisphere
- N_0 = 10 000 km in the Southern hemisphere

The width of a zone is about y 600 km

← Figure 4-12 presents the coordinates of the extremities of the zone are higher in Y than along the central line due to the curve of the parallels.

For any point coordinate, you must specify which zone you are referring to:

Example: UTM30 (56.012 , 125.236)

The French territory is located on 3 zones:

- UTM North zone 30 : between 6 degrees West and 0 degrees (= Greenwich). EPSG :32630
- UTM North zone 31 : between 0 degrees and 6 degrees East. EPSG :32631
- UTM North zone 32 : between 6 degrees East and 12 degrees East. EPSG :32632

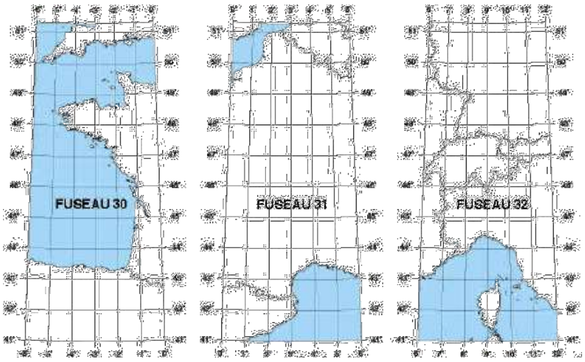


Figure 4-13 the 3 UTM zones of France ("fuseau" in French) (IGN)

EPSG codes: 326xx where xx is the number of the zone. Example zone34 = EPSG :32634

Distance distortion in UTM

No representation can keep all the lengths on the whole represented domain: the sphere (or the ellipsoid) cannot be "flattened" without deformations, each length will undergo an alteration which will depend on its position on the earth.

Step 1 : reduce to ellipsoid : elevation correction factor

As all coordinate systems come from the ellipsoid, the first step is to project the terrain measures to the ellipsoid.

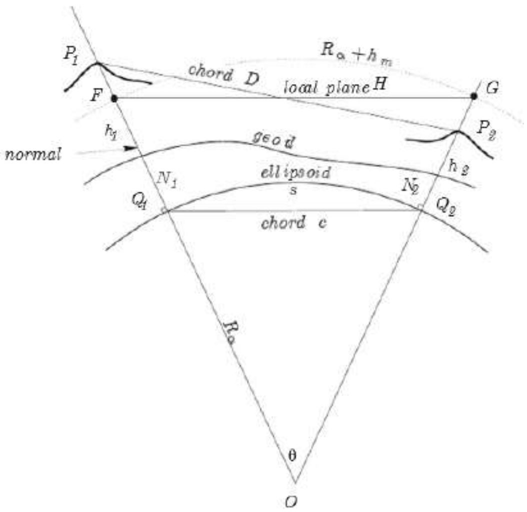


Figure 4-14 distance projection from field to ellipsoid

Measured distance between P1 and P2 = D

Vertical angle between P1 and P2 = V

Ellipsoidal height of P1P2 local plane = h (average h1 h2, see "elevation measurement" below for the definition of ellipsoidal height)

Local sphere radius Rn (in France Rn=6380km)

Flattened on local plane: H = D.sin V

Elevation correction factor EF= Rn/(Rn+h)

Reduced to the ellipsoid: c = D.Rn/(Rn+h)

Usually for topography we equate the chord distance to the bow distance (theoretically the bow distance has to be used...).

Example : In Nîmes, France, the ellipsoidal height is around 95m so the distances are modified by

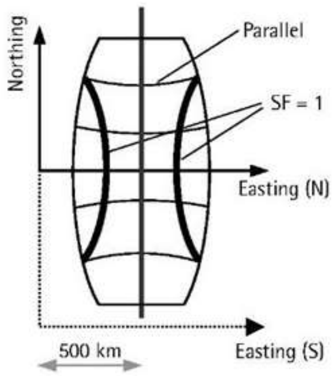
6380000/(6380000+95) = 0.999985

150m on the ground becomes 149.9978 m a loss of 2.2 mm

This projection to the ellipsoid is mandatory before any other projection for ALL systems.

Step 2: apply UTM projection scale factor

The relative variation of the lengths in the representation is called distance distortion, where a Scale Factor on distances has to be applied between the projection on the ellipsoid and the projection on the UTM stripe.



Here the SF=1 shows the standard meridians:

No distortion along them

Figure 4-15 scale factor=1 along standard meridians (what-when-how.com)

In conformal projections like UTM, the distance distortion is independent from direction.

In UTM a scale factor is applied: The formula to calculate the scale factor (k) at any geographic coordinate point (λ, φ) longitude latitude in relation to the longitude of the central meridian λ_0 is :

$$k \approx 0.9996\left(1+\frac{(\lambda-\lambda_0)^2}{2}\cos^2(\varphi)(1+\varepsilon^2)+\frac{(\lambda-\lambda_0)^4}{24}\cos^4(\varphi)\left(5-4\cdot\tan^2(\varphi)\right)\right)$$

With $\varepsilon^2 = \frac{a^2-b^2}{b^2}\cos^2(\varphi)$ (reminder WGS84 a = 6 378 137 m and b = 6 356 752 m)

On the central meridian, l=lo k = 0.9996 = -40cm/km any distance measured on between two points along the meridian (or very close) is multiplied by 0.9996 (or reduced of 40cm per km)

Example: 150m on the ellipsoid are 150x0.9996=149.94 m on the UTM plan (or 150-0.40x0.150),

It is shorter to 6 cm !

On the edges of the UTM zone along the limiting meridian, the scale factor is k = 1.00097 = +97cm/km

Example still for 150m on the ellipsoid are 150x1.00097=150.146 m on the UTM plan

(or 150+0.97x0.150), it is longer to 14.6 cm !

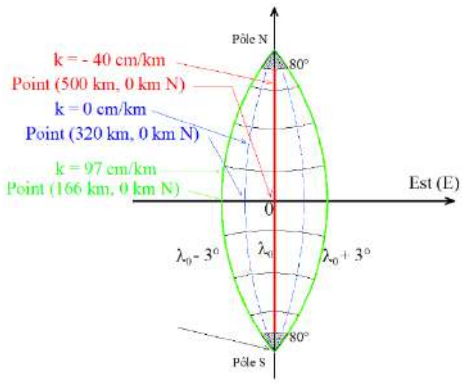


Figure 4-16 remarkable scale factors in UTM

In UTM, at constant latitude, the linear modulus of the projection increases when moving away from the central meridian, reaches 1,then becomes greater than 1. Along a meridian, the linear modulus decreases as the latitude increases.

Global distance distortion: ellipsoid reduction + projection distortion

4.5 Lambert's conical projection: in France, Belgium, Texas...

Lambert projection on a cone

The Lambert projection is a direct conic secant projection: the projection is made on a cone secant to the ellipsoid along two parallels.

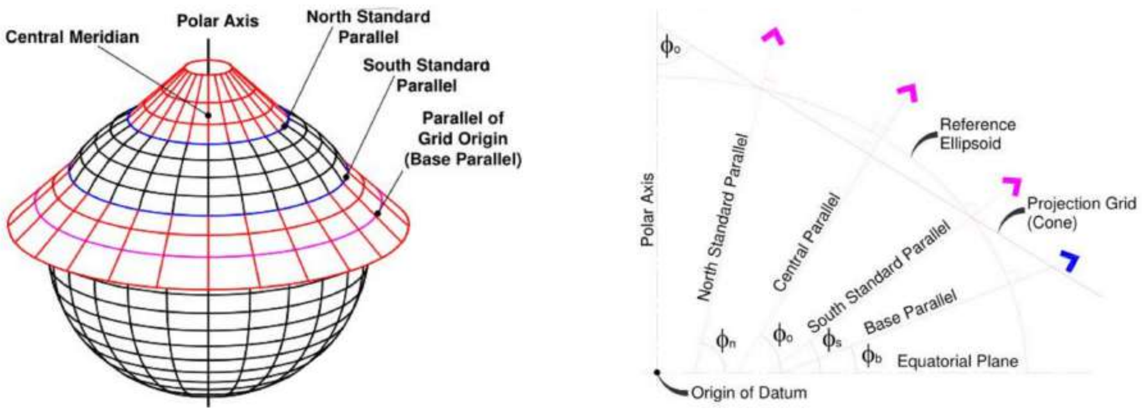


Figure 4-17 Lambert secant projection (researchgate.com)

Conform projection: preservation of angles, angles measured on terrain are the same on the Lambert map. Distance distortion.

On the map, the tangent with the cone parallels (= standard parallel) is represented by a circular arc.

The parallels have as image concentric circles. The irregular spacing of the parallels ensures the conformity of the representation.

The original meridian has as image on the map: the North axis (N).

The meridians have as image straight lines intersecting at the image of the North Pole P. They are therefore perpendicular to the concentric circles, and on maps we observe a meridian convergence.

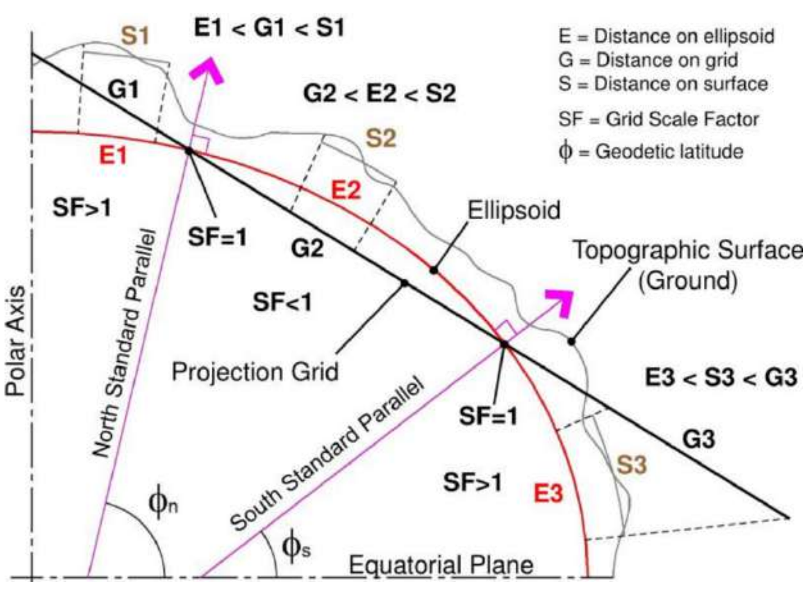
Reference ellipsoid : IAG GRS 80 (=WGS84)

Distance distortion in Lambert projection

In order to reduce the linear alterations the cone cuts the ellipsoid twice: direct secant and conformal conical projection. Along the secant parallels (cone/ellipsoid contact) there is no distance distortion (k=1).

To reduce distance distortion, France for example uses 2 Lambert systems:

- The GRF93-Lambert 93 that covers the entire territory and has large distance distortions
- The RGF93-9 zones in which the territory is divided into 9 zones, or as many Lambert projections with different values and less distance distortion. Each one has its own cone.



the on-site measured distances are S1, S2, S3

Step 1: projection to ellipsoid on E1, E2, E3

Step 2: projection on the cone gives G1, G2, G3; the distances between the coordinates in Lambert, and the drawing on the plan or map.

On the 2 standard parallels there is no distortion due to Lambert, but there is a distortion due to "step 1", the ellipsoid reduction.

Figure 4-18 different scale factors depending on position from standard parallels (Bryan W. Bunch)

North to the northern standard parallel and South to the southern standard parallel, the distortion is positive, or k>1, distances are bigger on the map than on the terrain.

Between the southern standard parallel and the northern standard parallel, the distortion is positive, or k<1, the Lambert plan is "inside" the ellipsoid, the distances on the map are smaller than on the terrain.

Convergence of meridians in Lambert representation

In most projections, the North of the map does not indicate the direction of the geographic North Pole. In this case it is a convergence of the meridians "c" which is, at a point, the bearing (c > 0 or c < 0) of the image of the meridian (at the time of the projection) which passes through this point.

In Lambert projection, the convergence "c" of a meridian at a point A varies according to the longitude λ of this point (with respect to the original meridian of longitude λ_0 but it is constant for all the

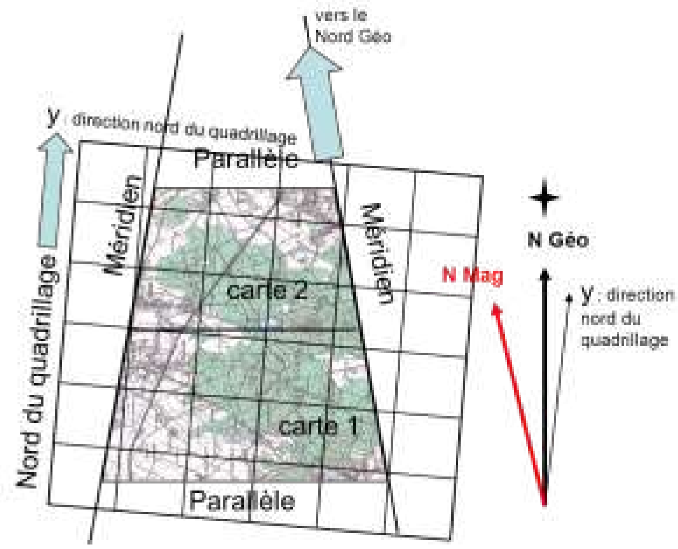


Figure 4-19 meridians convergence in Lambert (IGN)

points belonging to the same meridian (i.e. for λ = cte and $\forall\varphi$) :

<u>Note</u>: - convergence "c" is negative east of the prime meridian.
- the convergence "c" is positive to the West of the prime meridian.

Of course, if the map is centred on the prime meridian, then the north of the map is true north.

Geodetic system RGF93, Projection Lambert 93. Code EPSG:2154

The so-called Lambert 93 projection associated with the RGF 93 geodetic system has been the official projection for metropolitan France and Corsica since 2000, compatible with WGS 84 (GNSS) and the European ETRS89 system.

The reference ellipsoid of the Lambert 93 projection is the IAG GRS 80 ellipsoid (=WGS84) and the prime meridian is the meridian located at 3°E of the Greenwich meridian.

The Lambert 93 projection covers all of France and Corsica, it is secant in two parallels.

To locate points and draw plans or maps, we placed on the flattened cone an orthonormal reference in meters, its point of origin has the coordinates

E0 Lambert 93 = 700 km, N0 Lambert 93 = 6 600 km

The main characteristics of the Lambert 93 projection are :

- **North axis or prime meridian: λ_0 = 3° East / Greenwich**
- **Coordinates of the origin: E_0 = 700 km and N_0 = 6 600 km**
- **Prime parallel: φ_0 = 46° 30′ N (tangent to East axis in E0, N0)**
- Latitude of standard parallels : φ_1 = 44° N et φ_2 = 49° N
  - Central distance distortion : k=0,999 051 03 = -94,9 cm/km
- code : EPSG:2154 ou EPSG 5698 (with IGN69 altimetric grid)

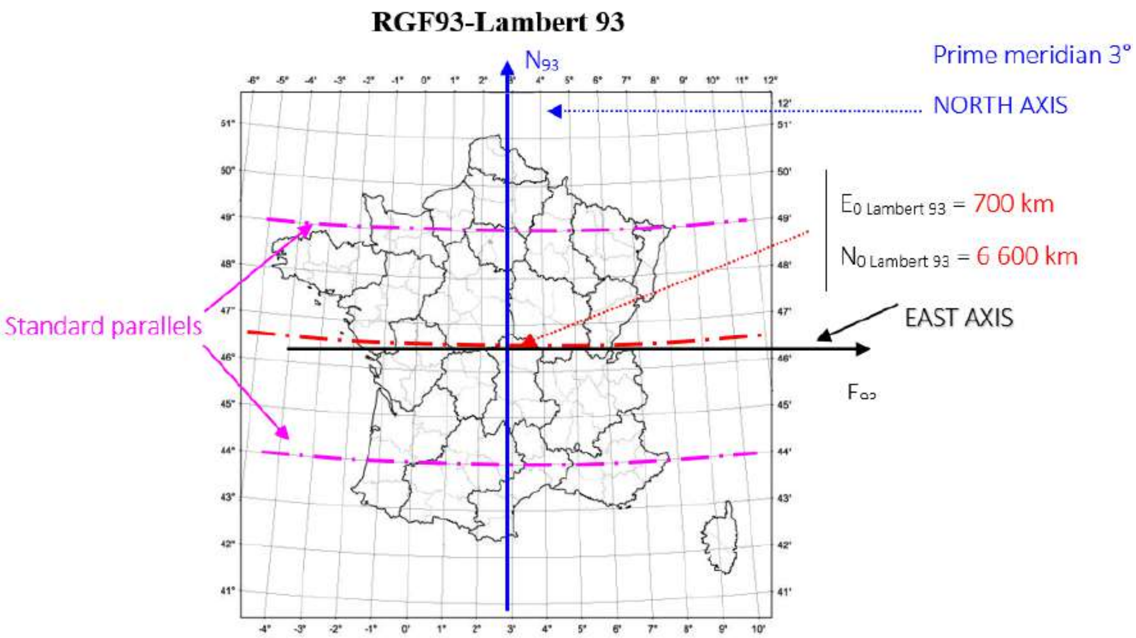


Figure 4-20 axis of the RG93-Lambert93 projection system

To easily recognise a point in Lambert93, its coordinates are:

East between 100 km and 1 200 km ; North between 6000 km and 7100 km

No confusion with the RGF93-9 zones coordinates (see below).

Distance distortion in Lambert 93

The map below gives the value of the distortion according to the latitude, there is no variation in longitude, unlike the UTM, Lambert is nice to surveyors.

We can see that in the North, the alteration goes up to 2m to be added on the map to the field measurements per kilometre measured to draw the plan!

In the centre it can go up to almost 1m to remove from the field measurements per measured kilometre to draw the map.

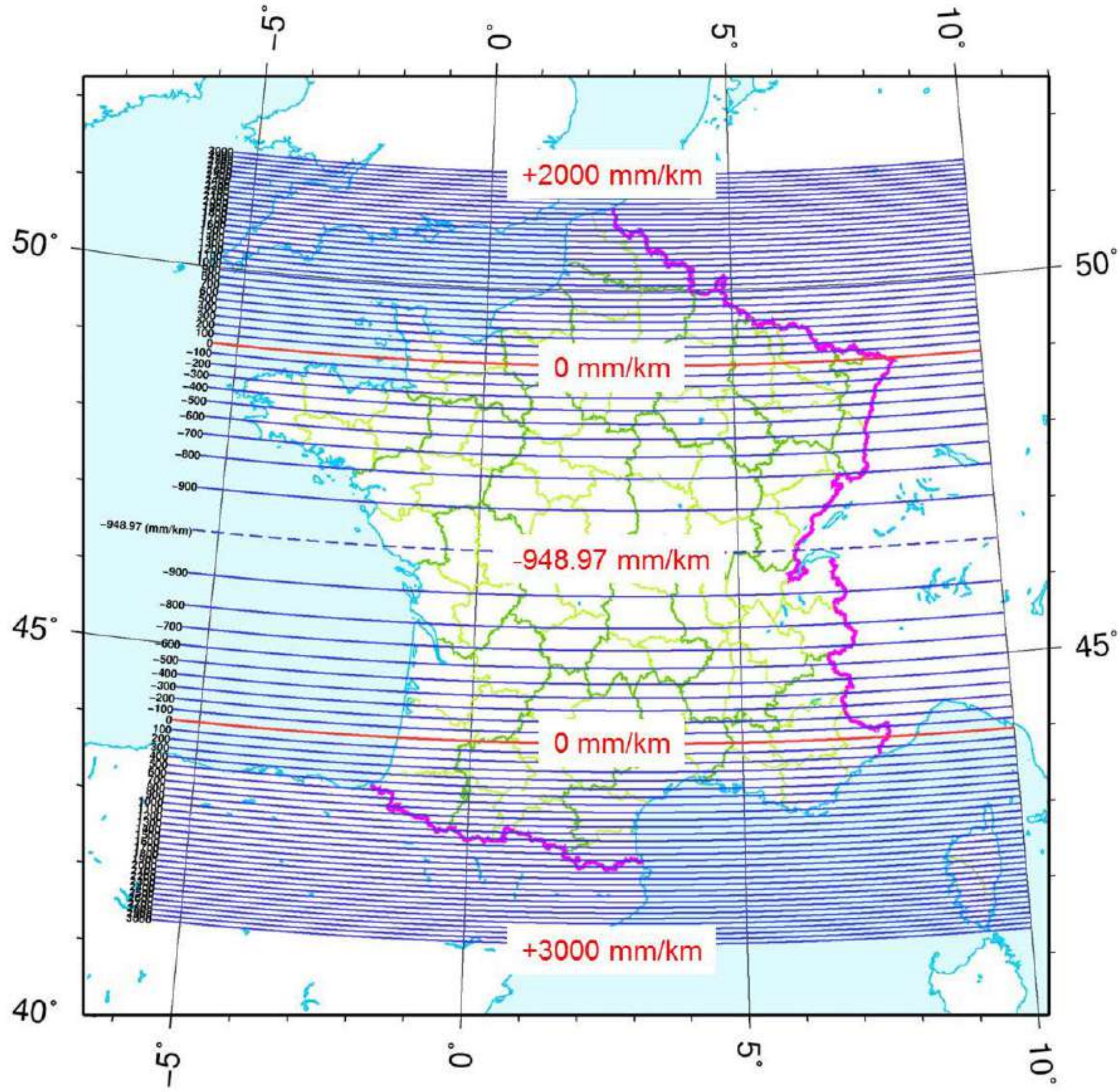


Figure 4-21 distance distortion in Lambert 93 (IGN)

Computation of the L93 scale factor using the distance from the central parallel (46.500°)

On the central parallel : k = – 949 mm/km

On any other parallel which distance from the central parallel (46.500°) is “d” in km

we obtain **k ≈ 1+ $d^2/(2.Rn)$ - $94.9.10^{-5}$**

With Rn = 6380km local sphere radius in France

Example for Dunkerque, 500 km north from the 46.500°

$$k \approx 1 + \frac{d^2}{2R^2} - 94{,}9 \text{ cm/km} = 1 + \frac{500^2}{2 \times 6380^2} - 94{,}9\ 10^{-5} = 1{,}002121 = 212 \text{ cm/km}$$

Point at the latitude of	Coordínate N en Lambert 93 (en km)	Distance distortion k
Dunkerque	$N = 7\,100 = N_0 + 500$	k = 1,00212 = + 212 cm/km
Meaux	$N = 6\,878 = N_0 + 278$	k = 0 car $\varphi = \varphi_0 + 2{,}78$ gon $= \varphi_2$
Poitiers	$N = 6\,600 = N_0$	k = 0.999051 = – 94,9 cm/km
Bonifacio	$N = 6\,000 = N_0 - 600$	k = 1.00347 = + 347 cm/km

The graph below can also be used to determine the distance distortion k of a location when its latitude is known φ.

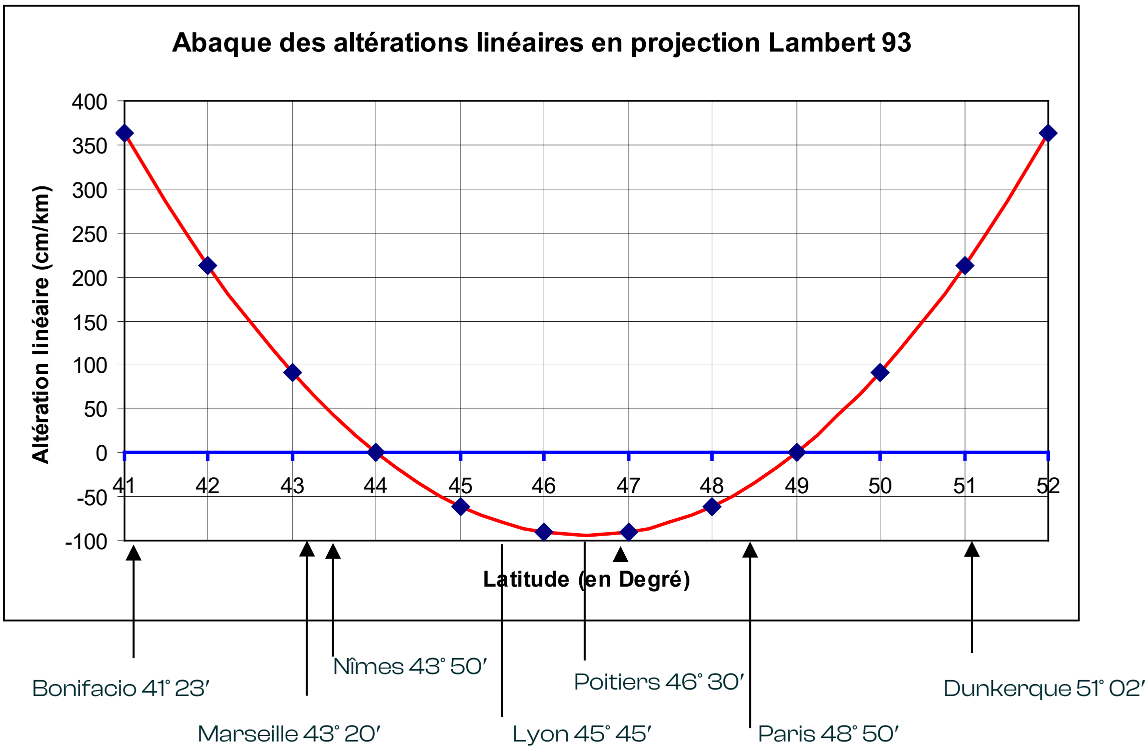


Figure 4-22 distance distortion in Lambert 93 (IGN)

In France, official IGN maps use only RGF93-Lambert 93 system, it is mandatory.

Example of LAMBERT 93 distance distortion for the zone « Bourg-en-Bresse to Troyes »

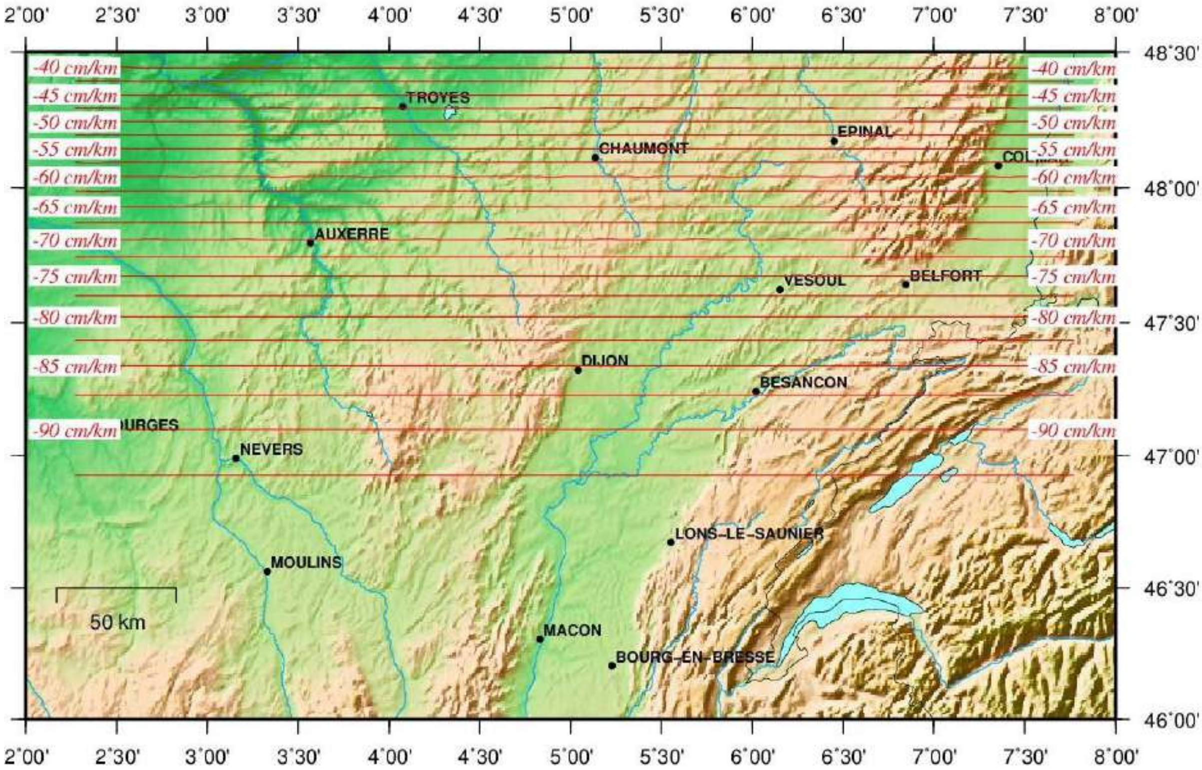


Figure 4-23 close-up around Dijon for distance distortion in Lambert93 (IGN)

Conical conformal 9 zones projection Codes EPSG:3942 (zone 1/CC42) à EPSG:3949 (zone 8/CC49)

To compensate for the low use of the Lambert 93 projection by professional land surveyors, due in large part to a distance distortion considered too strong, it was decided to create, in 2006, 9 additional projections to the single Lambert 93 projection.

The linear alteration in each of these 9 areas is always less than 10 cm/km.

This is less than 1cm to remove on the field measurements of the order of 100m maximum that we usually encounter.

- The main characteristics of the Nine Zone projections are :
- These 9 projections are Lambert Conformal Secant projections bearing the denomination CCxx (where xx is the latitude in degrees of the original parallel).
- Each of the 9 zones extends over 2 degrees of latitude and has for central parallel, a parallel whose latitude is an integer number of degrees.
- The "band" covered is about 222 km width (+111 north of the central parallel, -111 km south)
- The 9 zones cover the entire national territory with a 50% overlap between them (each department can thus be associated with a single projection).

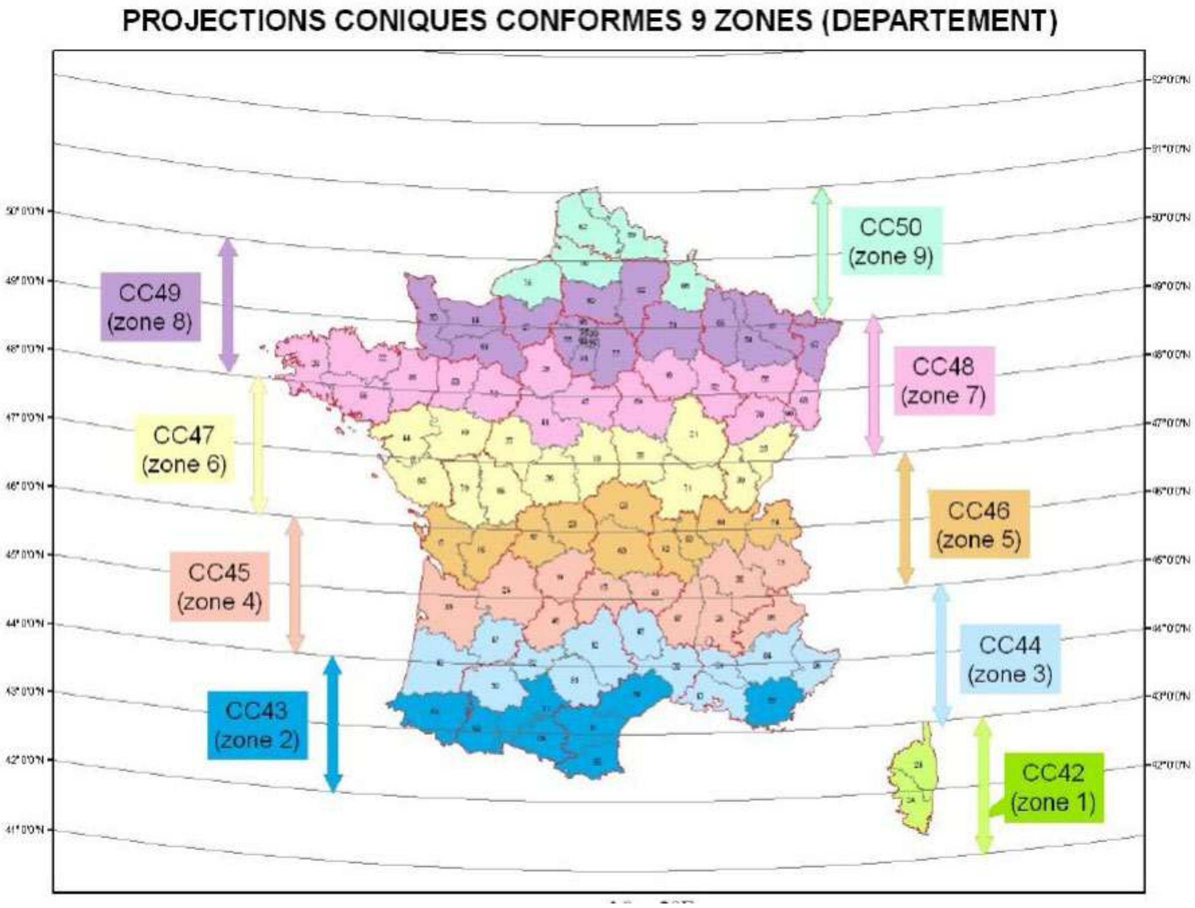


Figure 4-24 the 9 zones of RGF93-CC 9 zones projections (IGN)

For instance the **CC43** designates the conformal conic projection of the central parallel located at 43 degrees North, with a range extending from parallel 42° in the South to parallel 44° in the North.

We can therefore summarise the characteristics of the Nine Zones (NZ is the zone number) as follows :

- central meridian : λ_0 = 3°Est Greenwich,
- Latitude of central parallel: φ_0 = (41 + NZ)° N,
- Latitude of standard parallels: $\varphi_1 = \varphi_0 - 0{,}75°$ N et $\varphi_2 = \varphi_0 + 0{,}75°$ N,
- validity Zone: φ_0 +/– 1° = φ_0 +/– 111 km,
- Coordinates of the origin point : E_0 = 1 700 km et N_0 = (NZ × 1000 km) + 200 km.

Exemple : projection **CC44** (where Nîmes is) :

- zone number: NZ = 3
- Latitude of central parallel : $\varphi_0 = (41 + 3)° = 44°$ N,
- Latitude of standard parallels: $\varphi_1 = 44 - 0{,}75° = 43{,}25°$ N ; $\varphi_2 = 44 + 0{,}75° = 44{,}75°$ N
- validity zone: 43° N à 45° N,
- origin point : $E_0 = 1\,700$ km ; $N_0 = 3 \times 1000$ km + 200 km = 3 200 km.

The coordinates of a point in zone CC44 will therefore be of the following type, in metres:

(E_{CC44} 1 200 000 to 2 300 000m , N_{CC44} 3 100 00 to 3 300 000m)

Distance distortion in Lambert 9 zones

The distance distortion of each zone is less than 10 cm per km, which is about 2 times less than the alterations generated by the old Lambert zone and 30 times less in the extreme case of Lambert 93.

The linear alteration is between the values: - 9 cm/km < k < + 7 cm/km

The 9-zone projections are of interest for work on maps and paper plans for which a high accuracy is desired, their use is not justified for plans with an accuracy lower than the distance distortion.

Example of linear alteration in LAMBERT 93 CC47 for the same area as in Lambert 93 "Bourg-en-Bresse to Troyes

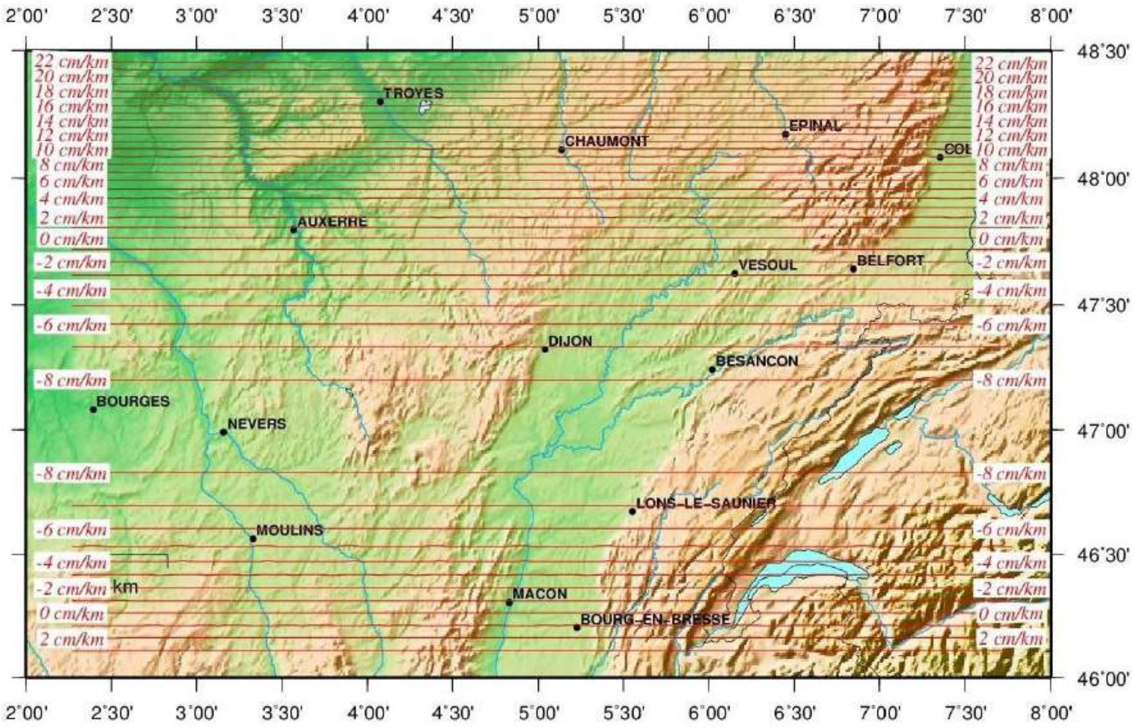


Figure 4-25 close-up around Dijon of distance distortion in RGF93-CC47 projection (IGN)

4.6 Geodetic benchmarks

In the filed one can find various forms of reference points, which can be found on the website

https://geodesie.ign.fr/fiches/index.php?module=e&action=visugeod

It is possible to place a prism pole or tripod or scale on some of them to check an instrument for example.

Figure 4-26 concrete and bronze geodetic benchmark in France (IGN)

5. Elevation measurements: the geoid

Across the world, elevations are measured above the mean sea level around the country.

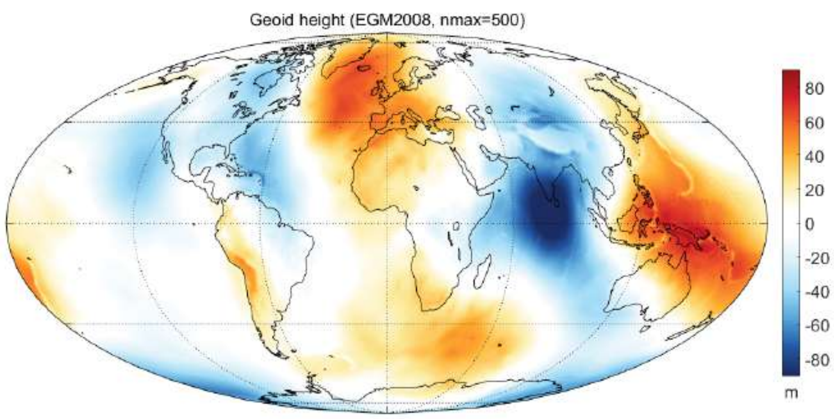
The ellipsoid is NOT the mean sea level (MSL), because this level depends on gravity, we need another reference for the MSL: the geoid.

5.1 The terrestrial geoid: different from the ellipsoid

The geoid is the MSL, which is extended under the continents by an equipotential of gravity allowing to determine everywhere an elevation. It differs from the ellipsoid, which is only used for positioning. The geoid passes above or below the ellipsoid, up to 100m!

The geoid has NO equation to determine it, it is only the result of hundreds of measures and observations it is continuously surveyed. (The ellipsoid is easy to calculate, has a mathematical equation).

Below the world geoid EGM 2008 and its detail in Europe and France (Earth Gravitational Model 2008)



World geoid EGM 2008 (2020 comming), position compared to the ellipsoid WGS 84.

The difference of height between the geoid and the ellipsoid varies from -107m (India) to +86m (Papua-New Guinea).

Figure 5-1 world geoid elevations compared to the ellipsoid WGS84 (Ales Bezdec)

In Europe, the geoid is between +10m and +60m above the ellipsoid

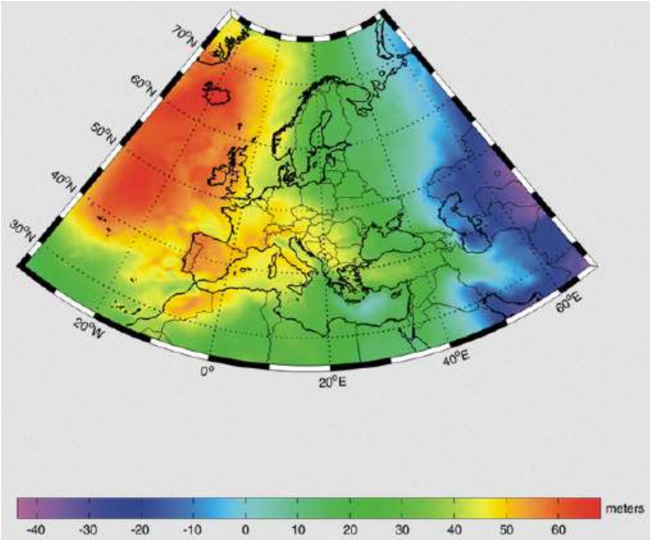


Figure 5-2 Europe's geoid (H. Denker, W. Torge)

France RAF18 geoid

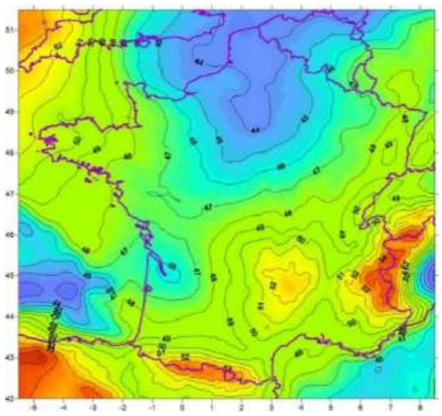


Figure 5-3 French geoid (IGN)

5.2 The geoid: determining the elevation of a point

If we measure elevations in relation to the ellipsoid we make an error of several tens of meters!

The elevation of a point is the distance from that point to the zero level surface (geoid) along the physical vertical through that point.

The geoid of each country passes through its height reference point "zero"; the mean sea level for the whole country.

Depending on the reference we take, one can measure the elevation either in relation to the ellipsoid or to the geoid. Only the geoid reference is the "true" elevation.

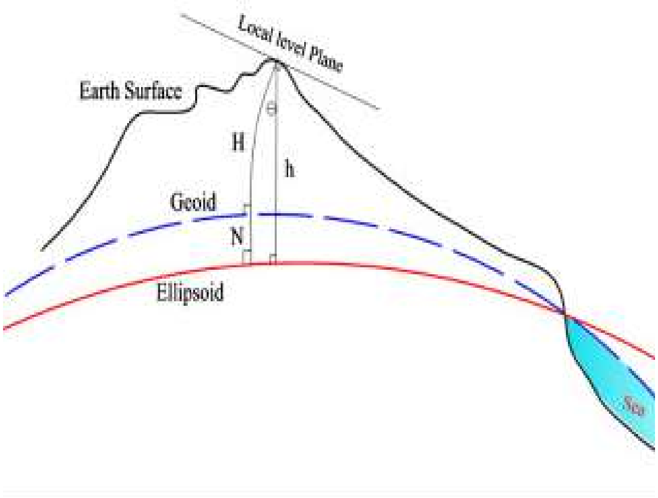


Figure 5-4 heights measurements (USGS)

h = ellipsoidal height, (measured by the GPS) perpendicular to the ellipsoid

H = normal height or orthometric height (that of the topographic plans and maps) following the local vertical of gravity, disturbed by the geology of the ground, perpendicular to the geoid and its equipotential. This is the "true elevation" or legal elevation.

N = geoid undulation (geoid-ellipsoid distance, given in the EGM2008 or RAF 18 grid) perpendicular to the ellipsoid at the point of projection of the earth point along the path of gravity (see below)

The normal height is also called the "legal elevation", to obtain it: **H = h - N**

5.3 Legal altimetry system

Each country has a legal system referring to the geoid of its country, passing through a reference point where a tide gauge is positioned defining elevation zero for the country. (for countries by the sea... for countries without seafront, they refer to a point of another country by the sea),

In France it is the NGF IGN 69 system. NGF = general levelling of France, the fundamental point is in Marseille, at the Marégraphe (tide gauge). The normal height: altitude defined by the IGN (French Geographical Institute) since 1969 with g = value of the so-called normal gravity field calculated for a theoretical ellipsoidal earth.

Figure 5-5 French levelling benchmark (IGN)

Geodetic benchmarks, or "levelling benchmarks" are located all over the country, they can be found on the website https://geodesie.ign.fr/fiches/index.php?module=e&action=visugeod

All benchmarks indicates the altitude in relation to the geoid, with an accuracy depending on their order, there are 4 orders in France.

The first order benchmarks, recognizable because their numbering consists of only 2 letters and a number, are the most accurate. For example U'M-39 at Lunel gives 6.549m (as there were not enough 26 letters, "prime" was added).

The first order benchmarks are accurate to each other at 2mm/km$^{1/2}$

The 4th order benchmarks are accurate at 3.6mm/km$^{1/2}$ between them.

6. Gnss satellite navigation system

6.1 Actual systems

The GNSS « global navigation satellite system » is composed with 3 elements :

- The space segment, which is composed of satellites rotating at about 20,000 km from the earth
- The control segment composed of ground stations that regulate the operation of the system
- The user segment, which is a receiver that computes its position

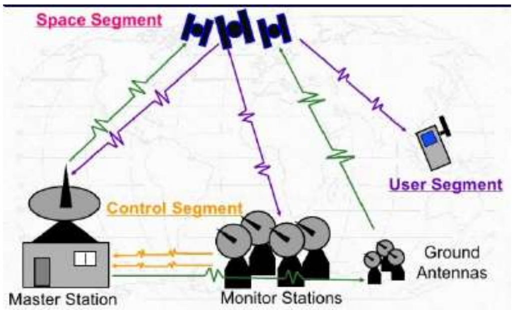


Figure 6-1 the 3 segments of GNSS (F.Burks)

Until 2007, only the GPS (Global Positioning System) -designed, developed and maintained by the US Department of Defense- was an operational GNSS. Since then, Glonass (USSR and then Russia) has arrived, followed by two other systems: the Chinese Compass/Beidou and Galileo of the European Union. Japan (QZSS) and India (IRNSS) are the last ones.

Figure 6-2 the 6 different GNSS (unknown origin)

GNSS orbits around 20 000 km and counts around 30 available satellites

GNSS	GPS	GLONASS	GALILEO	COMPASS (Beidou2)
Country				
Satellites + Spare (Plan)	27 + 3 (1993)	21 + 3 (2012)	26 + 4 (201x)	30 + 5 GEO (2015)
Satellites in Constellation	31 (2009)	19 (2009) 24 (2012) 3Y	2 (2009) 4 (2011) 2Y 18 (2013) 4Y	2(2009) 12 (2011) 2Y 30 (2015) 6Y
Orbital height	20180 km	19100 km	23222 km	21500 km
Orbital period	11:58 h	11:15 h	14.05 h	12:35 h
System Control	Military	Military	Civil	Military
Timing Services	Yes	Yes	Yes	Yes
Clocks	Cs, Rb	Cs	PHM, Rb	Rb
TimeScale	TAI-19	UTC-3 hours	TAI	
Time Offset transmission	GGTO GPS/Galileo Time Offset		GGTO GPS/Galileo Time Offset	
Open service / 95%	100 ns	100 ns	30ns	50ns
Open service / 95%	28m		35m	50m

Table 1 GNSS characteristics

6.2 Computation of the position

3D multilateration (positioning) : intersection of 3D spheres

The operation of GNSS is based on the measurement of the propagation time of the signal emitted by a satellite until it is received by the user. By multiplying this time by the speed of propagation of the wave we obtain the distance between the satellite and the antenna. The measurement of the propagation time of the signal from several satellites allows by spherical multilateration to determine the position of the receiver.

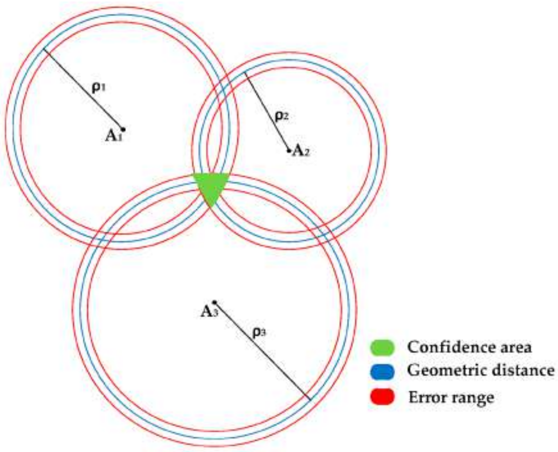


Figure 6-3 Multilateration in a plan (topomaths)

If we know the position A1, A2, et A3 (les satellites)

And if we know the distance between each Ai and the receiver, with its confidence, then the receiver (the user with his GNSS antenna) is in the central triangle of confidence, the only place respecting the 3 distance conditions.

Spatial multilateration: 4 satellites are needed to determine the intersections of spheres and not circles, but also to obtain a synchronization of the clocks between receiver and satellites.→

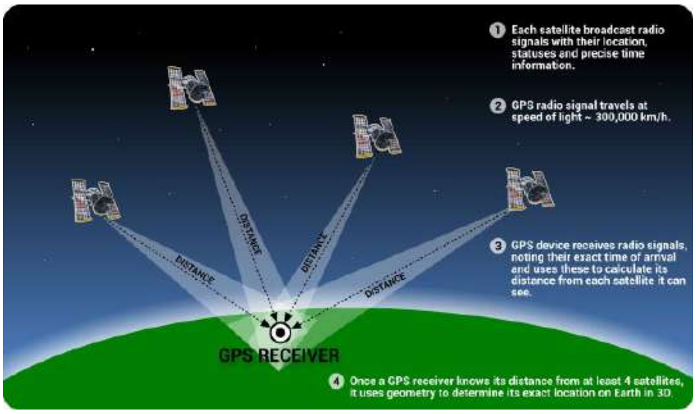


Figure 6-4 computation of a position from 4 satellites (geneco.rs)

The determined position cannot be a single point because of the confidence in the position of the satellites and the distance measurements: several calculations are made and an average is take

Positioning: exclusively on the ellipsoid WGS84

The reference ellipsoid is WGS84, GNSS can only position itself on this ellipsoid.

Longitude, Latitude, ellipsoidal height are the only information given by the receiver.

To obtain a legal altitude or the coordinates in the legal system of any country, it is necessary to use a software that will calculate these values using projection data and altimetric conversion grid.

6.3 GNSS native metric accuracy, GDOP concept, Kp index

Instant accuracy

The satellites emit electromagnetic waves (microwaves) towards the Earth, which propagate at the speed of light (300,000 km/s in vacuum). The receiver on Earth measures the time taken by the wave to reach its antenna (code shift, about 70 ms). It can then estimate the distance separating him from the satellite. With the distances to several satellites which position in known, the receiver can compute a spherical multilateration and obtain the position of the antenna.

The accurate measurement of this propagation time is crucial since an error of 10^{-6} seconds generates an error of 300 m on the distance.

Thus, an accuracy of 1 nanosecond (10^{-9}) is required to achieve a positioning accuracy in the meter range!

The positioning errors are due to the confidence on:

- the orbit of the satellites (known to within 5 to 30cm) obtained by the ephemeris(Table of values that gives the positions of astronomical objects in the sky at a given time or times) coded in the receiver and updated by the signal from the satellite and from the control segment on earth
- the synchronization of clocks between receiver and satellites (requires the nanosecond) obtained by decoding the received signal and atomic clocks in the satellites
- the propagation of the signal in the earth's atmosphere (ionosphere with its temperature, magnetic and electromagnetic fields, then troposphere with its humidity, pressure, temperature, and particles) which delays and alters the signal
- the parasitic reflections and masks in the receiving environment (high buildings, tree cover etc.) which delay and alter the signal
- the characteristics of the receiver antenna (variable position of the reception centre, phase centre, quality of the electronics, and number of frequencies received)

In instantaneous positioning, maximum precision in very good conditions:

- **10m with a single-frequency receiver** (cell phone, car GPS, hiking GPS etc., basic drone GNSS)
- **1m with a dual-frequency receiver** (professional antenna that processes 2 waves per satellite, some consumer devices do it too, rare and very expensive - high class drones)
- … Galileo (GNSS) proposes the reception of 4 frequencies to have, with an additional software, a centimetric precision, but it requires a very expensive quadri-frequency antenna.

This accuracy improves by staying static on the same point for a long time, GNSS makes the average of the measurements.

GDOP: Geometric Dilution Of Precision

This parameter, known in advance in the ephemeris of GNSS constellations, characterizes the positioning accuracy by the accuracy dilution factors, nDOP : Dilution Of Precision :

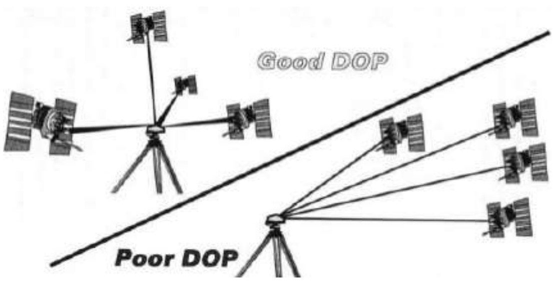
« n » is the different possibilities of DOP, σ0 is the measurement precision

- n = V : vertical solution : $VDOP = \frac{\sigma_h}{\sigma_0}$
- n = H : horizontal solution : $HDOP = \frac{\sqrt{\sigma_e^2 + \sigma_n^2}}{\sigma_0}$
- n = P : positioning solution : $PDOP = \frac{\sqrt{\sigma_e^2 + \sigma_n^2 + \sigma_h^2}}{\sigma_0}$
- n = T : Time solution : $TDOP = c\frac{\sigma_t}{\sigma_0}$
- **n = G : geometric and time solution :** $GDOP = \frac{\sqrt{\sigma_e^2 + \sigma_n^2 + \sigma_h^2 + c^2\sigma_t^2}}{\sigma_0}$

In general, only the GDOP is checked, which is the most complete data.
The larger the nDOP, the less accurate the result: for example, with a measurement accuracy of 10 meters and a GDOP of 7, the expected theoretical accuracy for an instantaneous positioning will be 70 m!

✓ **very good DOP <3**
✓ good 4-5
☹ not confident 6
☠ to avoid >6

↓ satellites well distributed over the measurement site will give a good DOP (nDOP < 3)



← poorly distributed satellites (or satellites with part of them hidden) will give a poor DOP (nDOP > 5)

Figure 6-5 DOP depending on satellites position (K.Ansari)

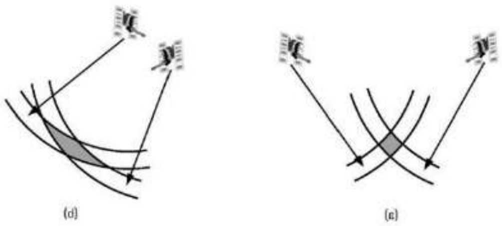


Figure 6-6 Distance intersections with their confidence area (marinegyaan.com)

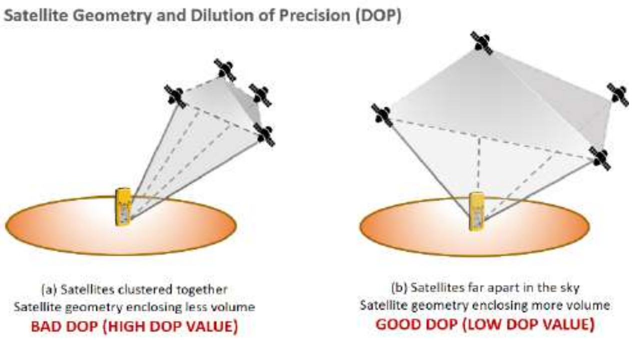


Figure 6-7 The closer the satellites are to each other, the wider the range of confidence (Graticule)

Variation during the day :

For 1 GNSS system and for 1 location, the DOP varies during the day, one can obtain a graph of the GDOP variations during the day, with a professional GNSS software.

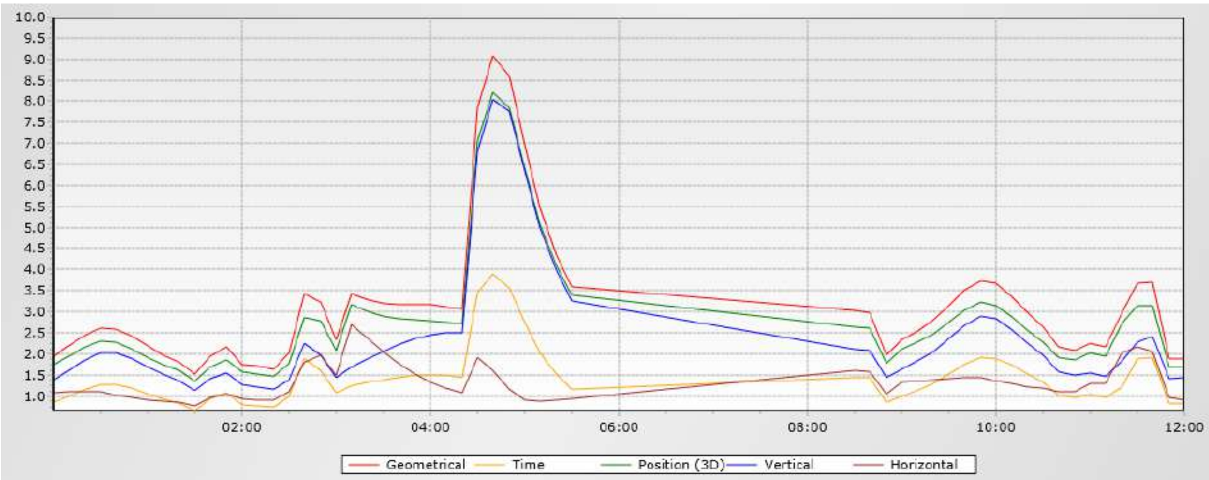


Figure 6-8 DOP variation during 12 hours (Trimble)

Above we see that the best moments of observation are between 8h30 and 9h30 then 10h30 to 11h15 finally around 11h45 (we can also work at night...).

WARNING : the beautiful peak around 5h00 am is the WORST TIME (big DOP).

Kp index

Kp index indicates the degree to which solar flares are disrupting the propagation of electromagnetic waves. It ranges from 0 to 9, and measures global geomagnetic activity. It. Up to 3, the activity is not very disruptive; at 4, it is significant; at 5 and above, GNSS satellite signals may be subject to strong variations, significantly reducing positioning accuracy.

There are weather applications and websites (ie. https://www.uavforecast.com/) that provide the Kp values for the tim e and location of planned UAV mission.

If precision is essential for the mission, try to fly with a **Kp < 4.**

6.4 Centimetric precision in PPP measurement mode

→ **PPP : precise point positioning** : a GNSS dual frequency antenna is left static for several hours or even days. It computes its position hundreds of times and by successive averages, the accuracy is in range of a centimetre.

It is not applicable for a drone survey: too long!

It is suitable however for a base station in differential post-processed mode (see below).

6.5 Centimetric range accuracy obtained with a differential system

Method: Linking two GNSS antennas

When linking two GNSS antennas, where one is fixed and its position is known (base station), and the other is taking measurements of points on the ground (rover station), the position of the measured points can be obtained to the centimetre.

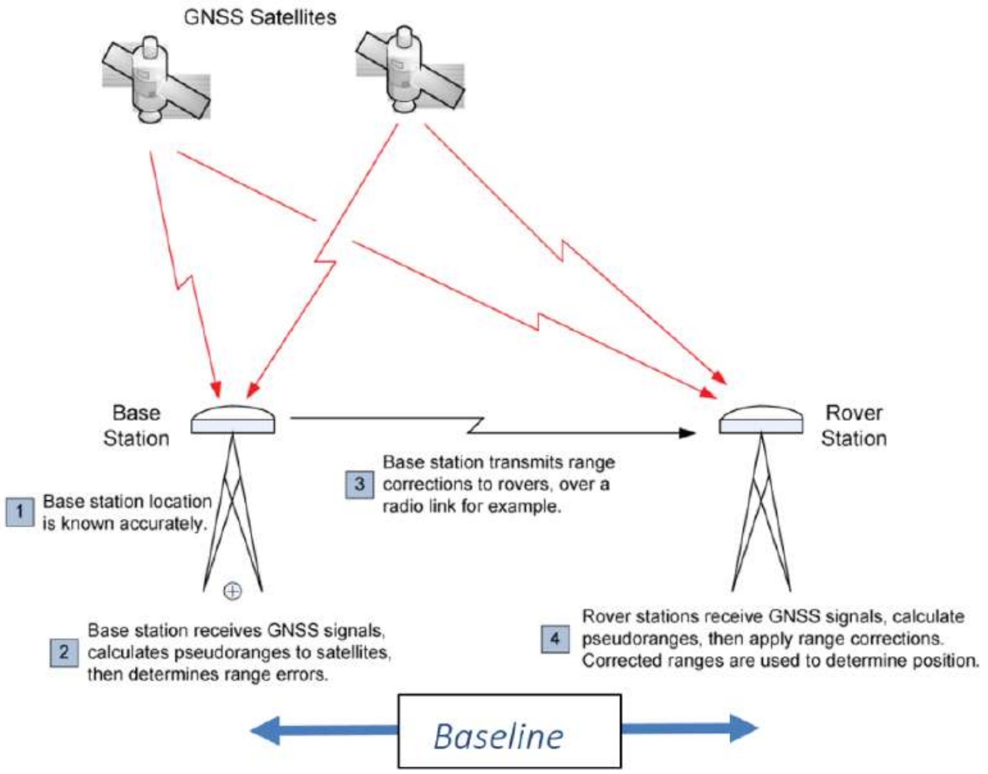


Figure 6-9 differential GNSS principle (Politecnico di Torino)

Post-processing base-rover: Setting up a fixed antenna in the field, the " base " which works as a reference, on a tripod, which acquires its position in static (a PPP of 1 to 2 hours), the points are surveyed with another antenna called " the rover ". A calculation is made a posteriori in the office to determine the position of the fixed antenna to the centimetre, and the coordinates of the points of the rover are then also adjusted to the cm by determining the errors to be corrected.

Baseline: distance between the known base station and the rover. The closer the better.

☺ best : < 10 Km
- ✓ suitable : 10-30 Km
  - ❖ Cautiously acceptable: 30 to 100 Km

☠ Not accurate > 100 Km

RTK: real time corrections via internet and subscription

RTK: Real Time Kinematic. It is a single antenna which is linked to a position correction server. This server calculates, thanks to a network of fixed antennas, the position with accuracy within 5mm. Corrections are to be made on the reception of all the satellites of the constellations surveyed. Thus the rover GNSS RTK calculates every second its precise position in the centimetre range.

In France, the RGP (permanent GNSS network) and various private antennas are used as a basis for several RTK fee-based services: Teria, Orphéon, Sat-info, Centipede.

To access RTK, operator needs:

- 4G or 5G modem on the GNSS decoder, in addition to the GNSS antenna, which is a different device
- A decoder compatible with RTK
- Subscribe (several hundred euros per year) to an RTK service
- Work in an area with cellular network coverage
- A SIM card (or a link with a phone via WiFi) to receive data

Steps to connect to RTK with all this equipment (it is assumed that the RTK configuration has already been done according to the subscription data of the RTK operator):

- Start GNSS: antenna and decoder
- Verify that the connection with the satellites is established and that a position is obtained
- Visualize on the screen the accuracy achieved : about a few meters
- Connect to the RTK via the dedicated menu
- While the connection is being established, within the settings,apply the measurement recording parameters
- Measurement recording only if **GDOP≤4**
- Automatic measurement recording if the **horizontal position accuracy ≤ 2cm** (to be adapted to the case, but below 1cm the measurement may take several minutes)

- Automatic recording of measurement if the **vertical position accuracy ≤ 2.5cm** (to be adapted to the case, but below 1.5cm the measurement may take several minutes)
- **Number of positions** calculated before automatic recording: **30** (so the GNSS has to make many determinations before recording, even if the accuracy is reached before, in case an error has occurred in the first calculations).
- Then set up the **cutoff angle** setting: satellites that are too low on the horizon are not taken into account, their signal crosses "too much" atmosphere" : set the angle to **15°** (or more in cities because of buildings that mask low satellites)
- In the main screen to check if the centimetre positioning quality is reached: **< 5cm**
- Start measurements

NB1: on the point display screen, the point whose name starts with RTCM is the antenna used as a reference, often several kilometres away, check the option "do not display RTCM" to have an easy zoom on all the surveyed points.

NB2: most RTK networks create virtual antennas, only for calculation purposes to avoid too much baseline.

Precision example for a Leica GS16 : standard deviation on one measure

Differential phase in post-processing

Type	Horizontal	Vertical
Static and rapid static	3 mm + 0.5 ppm	5 mm + 0.5 ppm
Kinematic	8 mm + 1 ppm	15 mm + 1 ppm
Static with long observations	3 mm + 0.1 ppm	3.5 mm + 0.4 ppm

Differential phase in real-time

Type	Horizontal	Vertical
Single Baseline (< 30 km)	8 mm + 1 ppm	15 mm + 1 ppm
Network RTK	8 mm + 0.5 ppm	15 mm + 0.5 ppm

Table 2 Leica GS16 antenna specs (Leica)

Example: if working in nRTK, with an RTCM virtual antenna at 20km (baseline=20km), the standard deviation σ on the measures is:

$$\sigma = \sqrt{8^2 + (0.5 \times 20)^2} = 13mm$$

- The accuracy of the vertical position is always about 1.5 times worse than the horizontal position

6.6 Local disturbances on the accuracy of a GNSS

Independent on the quality of the equipment and all atmospheric perturbations, there are local GNSS survey disturbances:

Multipath by facades or cliffs: the signal is delayed by a reflection on a facade which lengthens its path.

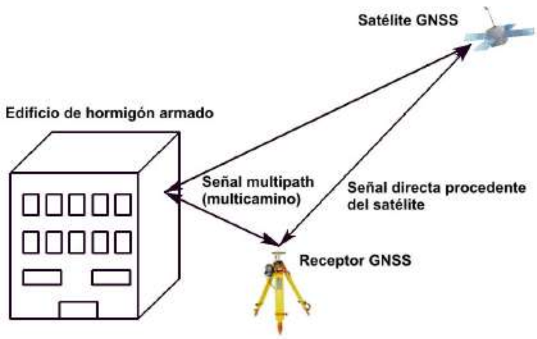


Figure 6-10 facade reflections (N.Garrido)

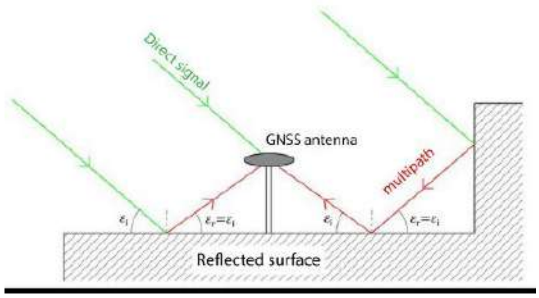


Figure 6-11 ground multipath reflection of waves from satellites

Ground multipath: guard ring antennas are used (choke ring)

Figure 6-12 Choke ring antenna (Trimble)

Mask: the signal is cut off by an obstacle between the receiver and the satellite: trees, storm clouds, buildings.

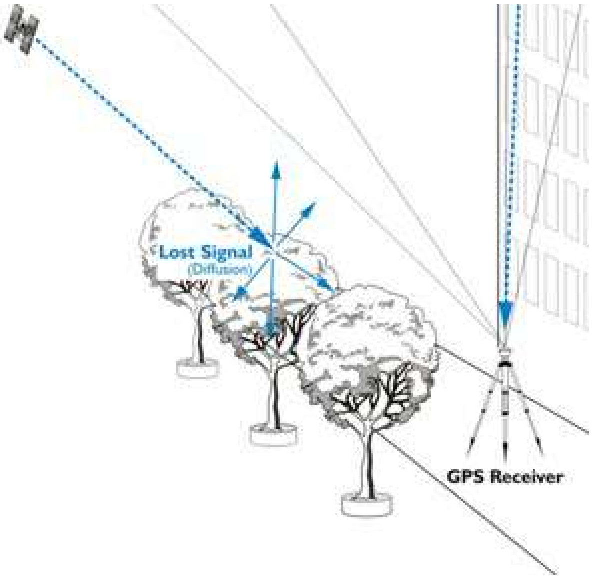


Figure 6-13 trees and buildings masking the signal (Penn state U.)

6.7 Legal system and GNSS

To link to a legal system, drone photogrammetry or LiDAR point clouds, there is necessity of the coordinates to be expressed in the legal system related to the site. Since GNSS is only positioned on the ellipsoid, it must be able to return coordinates that can be used for topography: legal coordinates (UTM or Lambert depending on the country, see above), and legal altitudes, relative to the mean sea level of the country, its geoid.

It is therefore necessary to have previously loaded in the decoder:

- Projection formulas between the ellipsoid and the legal system (ie.RGF93 in France, Lambert93 and CC 9 zones) to have the E and N legal coordinates. These formulas are mathematical and well known.
- An altimetric conversion grid (RAF20 grid the most recent in France, otherwise RAF18) to have the legal altitude, linked to the levelling marks of the country.

The math formula is very simple: $H = h - N$ the GNSS measures the ellipsoidal height "h" but the value of the undulation of the geoid "N" depends on the position of the measured point.

Each country establishes a grid, as fine as possible of its territory, of points where the height between the WGS84 ellipsoid and the geoid is determined (the undulation). The height reference grid is the list of coordinates of all points where the value of the vertical distance between the WGS84 ellipsoid and the geoid has been computed. In Europe this "undulation" is positive.

Between two points the undulation is interpolated.

The accuracy of altimeter conversion grids varies from 5 to 30 mm.

NB: if you forget to record the points in the right system, it is always possible to convert all the points afterwards with a software.

7. Geolocation targets on site

7.1 Principle (see also "photogrammetry" module)

When photogrammetry or LiDAR point clouds are to be geolocated, landmarks on the photos with position has been surveyed with a GNSS centimetric system (post-processing or RTK) is needed.

Targets are positioned in the field as wide apart as possible including large visible landmarks with numbers, and then surveyed with a centimetric GNSS system.

For a good geolocation it is suggested to have at least 4 targets. Of course placing more targets improves the accuracy.

7.2 Types of targets

Please see below for an example of a typical landmark target :

Figure 7-1 ground geolocation target (pixwing.fr)

The targets must be

- large (at last 50cm)
- numbered
- very stables (heavy or fixed by weights or stones)
- steady during all the drone survey
- well spaced out and well distributed in the workplace

7.3 Surveying the targets

The GNSS must be pointed to the exact centre of the target, stabilised by a tripod

The accuracy must be to the centimetre, the GNSS must be set before and connected in RTK (or base-rover with post-processing).

The number of the surveyed point must correspond to the number on the target.

The GNSS coordinate system must have been chosen before the survey, as well as the altimetric conversion grid.

Figure 7-2 GNSS measure on a target (Chivas)

List of figures

List of tables